MCF3M

Name: _____

EXPONENTIAL FUNCTIONS
UNIT REVIEW

50

1. Write as a single power. **Express answers with positive exponents.** (5 marks)

a) $3^4 \times 3^8 \times 3$ b) $((-9)^2)^5$ c) $\frac{5^6}{5^4}$ d) $\frac{4^7 \times 4^5}{4^{12}}$ e) $\frac{6^{10}}{(6^6)^2}$

$= 3^{4+8+1}$ $= (-9)^{10}$ $= 5^{6-4}$ $= 4^{7+5-12}$ $= 6^{10-12}$

$= 3^{13}$ $= 5^2$ $= 4^0$ $= 6^{-2}$

$= \frac{1}{6^2}$

2. Evaluate each expression **without using a calculator**. (5 marks)

a) $125^{\frac{1}{3}}$ b) $16^{\frac{2}{3}}$ c) $(2x^2y)^2$ d) $9^{-\frac{1}{2}}$ e) $\frac{2^2}{2^{-1}}$

$\sqrt[3]{125}$ $= \left(\sqrt[3]{16}\right)^2$ $= 2^2 x^{2\times 2} y^2$ $= \frac{1}{\sqrt{9}}$ $= 2^{2-(-1)}$

$= 5$ $= 4x^4y^2$ $= \frac{1}{3}$ $= 2^3$

$= 8$

Simplify each of the following. **Answers must have positive exponents.** (15 marks)

a) $(x^4)(x^3)$

$= x^7$

b) $(5x^8)(4x^{-1})$

$= 20x^{8+(-1)}$

$= 20x^7$

c) $(x^2y)^4$

$= x^8y^4$

d) $(y^2)^2 \times (y^3)^{-1}$

$= y^4 \times y^{-3}$

$= y^{4-3} = y^1$

e) $x^7 \div x^3$

$= x^{7-3}$

$= x^4$

f) $\frac{16x^5}{4x^2}$

$= 4x^{5-2}$

$= 4x^3$

g) $a^3 \times a^{-1} \div a^4 \times a^3$

$= a^{3+(-1)-4+3}$

$= a^1$

h) $\frac{(3x^4y^2)(15x^5y^3)}{9x^6y^7}$

$= \frac{45x^{4+5}y^{2+3}}{9x^6y^7}$

$= 5x^{9-6}y^{5-7}$

$= 5x^3y^{-2} = \frac{5x^3}{y^2}$

i) $\frac{(5x^4y^3)(4xy^2)}{10x^3y}$

$= \frac{20x^5y^5}{10x^3y}$

$= 2x^2y^4$

4. Calculate the differences for each table of values. Then use the first and/or second differences to classify each function as **linear, quadratic or exponential.** **(3 marks each)**

a)

x	y		
1	-1		
2	3	4	
3	9	6	2
4	17	8	2
5	27	10	2
6	39	12	2

quadratic

b)

x	y		
-1	0.25		
1	0.5	.25	
3	1	.5	.25
5	2	1	.5
7	4	2	1
9	8	4	2

exponential

c)

x	y	
-2	3	
-1	8	5
0	13	5
1	18	5
2	23	5
3	28	5

linear

d)

x	y		
-2	10		
-1	30	20	
0	90	60	40
1	270	180	120
2	810	540	360
3	2430	1620	1080

exponential

5. The value of a car after it is purchased depreciates according to the formula $V(n) = 35000(0.88)^n$, where $V(n)$ is the car's value in the *nth* year after it was purchased.

a) What is the purchase price of the car? **(1 mark)**

$35000

b) What is the annual rate of depreciation? **(1 mark)**

$P = P_0(1-r)^n$ $\quad 1-r = 0.88$ $\quad r = 1-0.88 = 0.12 = 12\%$

c) What is the car's value at the end of 3 years? **(2 mark)**

$V_{(3)} = 35000(0.88)^3$

$= \$23851.52$

d) How much value does the car lose in its first year? **(2 marks)**

$V_{(1)} = 35000(0.88)^1$

$= \$30800$

Value Lost $= \$35000 - \$30800 = \$4200$ lost

6. The population of a small town has increased at a rate of 1.5% per year since 1980. The town had a population of 2500 that year.

$P = P_0(1+r)^n$

a) Write the equation that models the growth in population of the town. **(2 marks)**

$P_0 = 2500$

$r = 1.5\% = 0.015$

$P = 2500(1+0.015)^n$

b) Use your equation to determine the population of the town in 2015. **(1 mark)**

$n = 2015 - 1980 = 35$

$P = 2500(1.015)^{35}$

$= 4209.70$

$= 4210$

7. The decay of a radioactive substance can be modelled by the equation below. **Assume mass is in grams and time is measured in minutes.** Determine:

$$A = 600\left(\frac{1}{2}\right)^{\frac{t}{25}}$$

→ total time

In 100

$A = 600\left(\frac{1}{2}\right)^{100/25}$

$= 600\left(\frac{1}{2}\right)^4$

a) The initial mass of the radioactive substance. (1 mark)

600

b) The time it takes for half of the substance to decay. (1 mark)

25 minutes

c) The amount of radioactive material remaining after 65 minutes (to one decimal place) (2 marks).

$A = 600\left(\frac{1}{2}\right)^{\frac{65}{25}}$

$= 600\left(\frac{1}{2}\right)^{2.6}$

$= 98.96 = 99.0\text{g}$