4.5 Using Quadratic Function Models to Solve Problems (P234 to241)

The table records the height of a rock launched from a slingshot, where time, t, is in seconds and height, h(t), is in metres.

Time (s)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Height (m)	2	10.75	17	20.75	22	20.75	17	10.75	2

a) Use the table to determine an algebraic model in **vertex form**.

$y = a(x-h)^2 + k$ from the table $V(2.0, 22)$

point $(0, 2)$

$2 = a(0-2)^2 + 22$

$2 = a(4) + 22$

$-20 = 4a$

$a = -5$ $\therefore$ $y = -5(x-2)^2 + 22$

b) Use the model to determine when the rock hits the ground.

$y = 0$ → rock hits the ground

$-5(x-2)^2 + 22 = 0$

$-5(x-2)^2 = -22$

$(x-2)^2 = -\frac{22}{-5}$

$x-2 = \pm\sqrt{\frac{22}{5}}$

$x = 2 + \sqrt{\frac{22}{5}}$ or $x = 2 - \sqrt{\frac{22}{5}}$

$x \doteq 4.1s$ or $x \doteq -0.1s$

$\therefore$ The rock hits the ground after 4.1s.

The population of a rural town can be modelled by the function $P(x) = 3x^2 - 102x + 25\,000$, where x is the number of years since 2000. When will the population be the lowest? Solve by **completing the square**.

$$P(x) = 3(x^2 - 34x + 289 - 289) + 25000$$

$$= 3(x^2 - 34x + 289) - 867 + 25000$$

$$= 3(x - 17)^2 + 24133$$

∴ The population (24133) is lowest in 2017.

A bus company usually charges \$2 per ticket. Revenue generated is given by $R(x) = -40(x - 5)^2 + 25\,000$, where x is the number of \$0.10 increases and the R(x) is in dollars. What ticket price should be charged if the company wants to earn \$21 000? Solve using the **quadratic formula**.

$$R(x) = -40(x-5)^2 + 25000$$

$$21000 = -40(x-5)^2 + 25000$$

$$21000 = -40(x^2 - 10x + 25) + 25000$$

$$0 = -40x^2 + 400x - 1000 + 4000$$

$$0 = -40x^2 + 400x + 3000$$

$$x = \frac{-400 \pm \sqrt{400^2 - 4(-40)(3000)}}{2(-40)}$$

$$x = \frac{-400 \pm 800}{-80}$$

$x = -5$ or $x = 15$

↓

means 15 \$0.10 increases

∴ Ticket price is $\$2 + 15(\$0.10) = \$3.50$

HW P239 #5, 6, 7, 9, 13, 14