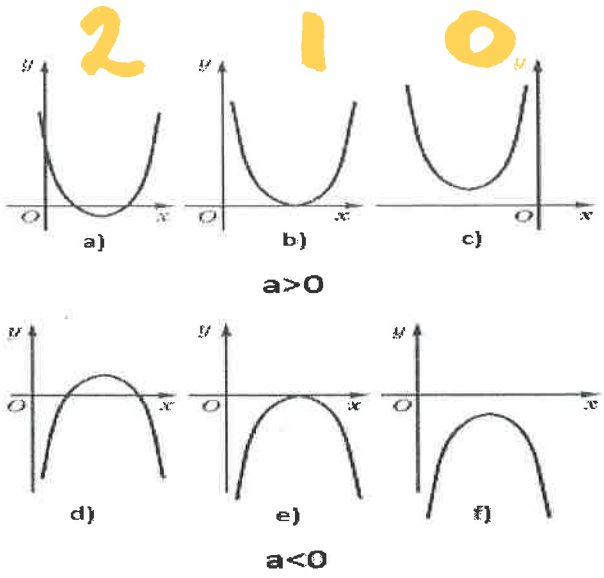
4.4 The Nature of the Roots (P227 to 233)

Quadratic equations can have 2, 1, or 0 real roots.



The value of the discriminant, $b^2 - 4ac$ (from the quadratic formula) tells us how many real solutions a quadratic equation has and how many x-intercepts the corresponding function has.

Examples

1. Factor and solve for the roots, then calculate then discriminant.

a) $x^2 - 8x + 12 = 0$

$(x-6)(x-2)=0$

$x=6$ or $x=2$

(two distinct real roots)

12

$-6 \quad -2$

$b^2 - 4ac$

$= (-8)^2 - 4(1)(12)$

$= 64 - 48$

$= 16$

$b^2-4ac > 0$

b) $x^2 - 8x + 16 = 0$

$(x-4)(x-4)=0$

$x=4$ or $x=4$

(two equal real roots)

$b^2 - 4ac = (-8)^2 - 4(1)(16)$

$= 64 - 64$

$b^2-4ac = 0$

2. Solve using the quadratic formula.

$x^2 - 5x + 11 = 0$

$$x = \frac{+5 \pm \sqrt{(-5)^2 - 4(1)(11)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{25 - 44}}{2}$$

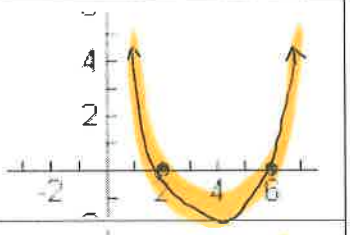
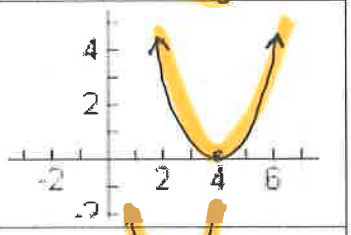
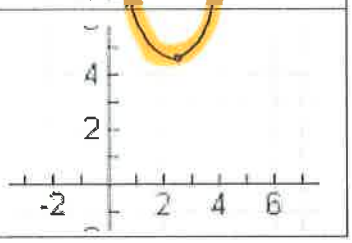
$$= \frac{5 \pm \sqrt{-19}}{2}$$

no real roots

$b^2-4ac = -19$

$b^2-4ac < 0$ no real roots

Summary

Number of Roots	Discriminant	Graph
2	$b^2-4ac > 0$	
1	$b^2-4ac = 0$	
0	$b^2-4ac < 0$ a negative	

3. For what value of k does the equation $8x^2 + 4x + k = 0$ have two distinct real solutions? One solution? No solution?

$a=8 \quad b=4 \quad c=k$

for 2 distinct roots, $b^2 - 4ac > 0$

$4^2 - 4(8)k > 0$

$16 - 32k > 0$

$16 > 32k$

$\frac{16}{32} > \frac{32k}{32}$

$\frac{1}{2} > k$

or $k < \frac{1}{2}$

$b^2-4ac=0$

One solution,

$\frac{16}{32} = \frac{32k}{32}$

$\frac{1}{2} = k$

$b^2-4ac < 0$

No solution,

$\frac{1}{2} < k$

$k > \frac{1}{2}$

check using graphing calculator

$8x^2+4x+0=0$

$8x^2+4x+\frac{1}{2}=0$

$8x^2+4x+1=0$

P232-233 #2ace, 4ace, 9, 12, 13