4.3 Solving Quadratic Equations using the Quadratic Formula (P216 to 223)

All quadratic equations of the form $ax^2 + bx + c = 0$ can be solved (for the roots, zeros, x- intercepts) using the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Derivation

By completing the square, in the general standard form, and isolating **x** the quadratic formula can be derived.

$$ax^2 + bx + c = 0$$

Step 1. Factor the coefficient $\quad a\left(x^2 + \frac{b}{a}x + \qquad \right) + c = 0$

Step 2. Complete the square $\quad a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c = 0$

Step 3. Group the first 3 terms $\quad a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c = 0$

Step 4. Factor $\quad a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0$

Step 5. Simplify $\quad a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - \frac{4ac}{4a}$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples

1. Solve each of the following using the quadratic formula, and graph.

a) $x^2 - 8x + 12 = 0$

$a=1 \quad b=-8 \quad c=12$

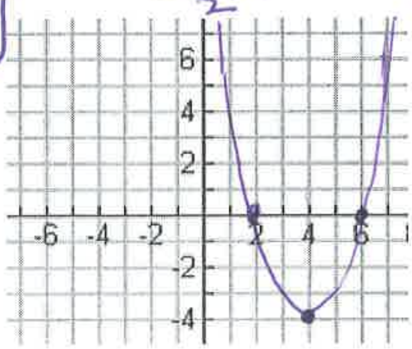
$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(12)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{64-48}}{2}$$

$$= \frac{8 \pm \sqrt{16}}{2}$$

$$= \frac{8 \pm 4}{2} \quad \therefore x = \frac{12}{2} = 6 \text{ or } x = \frac{4}{2} = 2$$

2 distinct real roots



(If $x=0$, $y=12$)

axis of symmetry $\frac{6+2}{2} = 4$

then $y = 4^2 - 8(4) + 12$
$= 16 - 32 + 12$
$= -4$
$(-4, 4)$ vertex

b) $x^2 - 8x + 16 = 0$

$a=1 \quad b=-8 \quad c=16$

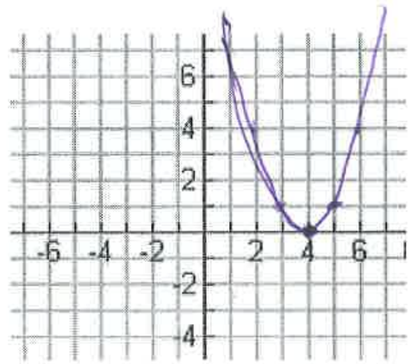
$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(16)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{64 - 64}}{2}$$

$$= \frac{8 \pm 0}{2}$$

$= 4 \Leftarrow$ 2 equal real roots

4,0 (vertex)



2. Solve using the quadratic formula.

a) $x^2 - 8x + 20 = 0$

$a=1 \quad b=-8 \quad c=20$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(20)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{64-80}}{2}$$

not possible

$\Rightarrow$ no Real roots

b) $2x^2 - 1 = 5x$

$2x^2 - 5x - 1 = 0$

$a=2 \quad b=-5 \quad c=-1$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25+8}}{4}$$

$$= \frac{5 \pm \sqrt{33}}{4}$$

$=$

HW P222-223 #1, 3abc, 4abc, 6, 8, 9