4.2 Relating Standard and Vertex Forms (P206 to 215)

A model rocket is shot from a platform 1 m above the ground. The height of the rocket above the ground is recorded in the table.

Time (s)	0	0.5	1	1.5	2.0	2.5	3.0	3.5	4.0
Height (m)	1.00	14.75	26.00	34.75	41.00	44.75	46.00	44.75	41.00

Vertex Form from a Table of Values

Looking at the table, give the coordinates of the vertex. $(3.0, 46.00)$ maximum height

Substitute the coordinates of the **vertex (h, k)** and one other **point (x, y)** into the vertex form and solve for **a**.

$y = a(x - h)^2 + k$

$(3, 46)$ and $(0, 1)$

h, k x, y

$1 = a(0-3)^2 + 46$

$-45 = 9a$

$a = -5$

$\therefore\ y = -5(x-3)^2 + 46$

Vertex Form to Standard Form

Expand and simplify the vertex form to get the standard form of the equation.

$y = -5(x-3)(x-3) + 46$

$= -5(x^2 - 6x + 9) + 46$

$= -5x^2 + 30x - 45 + 46$

$y = -5x^2 + 30x + 1$

1 2 3 4 5

Standard Form to Vertex Form - factor, calculate: zeros, vertex, and "a" value

factor a

$f(x) = -2x^2 - 8x + 24$

a $= -2(x^2 + 4x - 12)$

$= -2(x+6)(x-2)$

$-12 = (+6)\times(-2)$, $+6 + (-2) = +4$

zeros

$x = -6$ or $x = 2$

axis of symmetry $x = \frac{-6+2}{2}$

$\therefore x = -2$

$\therefore y = -2(x+2)^2 + 32$

vertex

$f(-2) = -2(-2)^2 - 8(-2) + 24$

$= -8 + 16 + 24$

$= 32$

$\therefore V(-2, 32)$

h, k

Standard Form to Vertex Form - Completing the Square

$f(x) = -2x^2 - 8x + 24$

1. $= -2(x^2 + 4x \quad) + 24$
2. $= -2(x^2 + 4x + 4 - 4) + 24$

$\frac{1}{2} \times 4 = 2^2 = 4$

3. $= -2(x^2 + 4x + 4) + 8 + 24$

$\therefore f(x) = -2(x+2)^2 + 32$ 4.

Steps:

1. Factor the coefficient of x^2 from the first two terms
2. Add and subtract the square of half the coefficient of x
3. Group the three terms that form the perfect square, multiply the fourth term by **a**, and move it outside the brackets
4. Factor the perfect square and simplify

Examples:

1. State the max / min value by completing the square for each of the following.

a) $y = x^2 - 6x + 8$ $\frac{6}{2} = 3^2 = 9$

$= (x^2 - 6x + 9 - 9) + 8$

$= (x^2 - 6x + 9) - 9 + 8$

$y = (x - 3)^2 - 1$

$\therefore$ min. of -1

b) $y = -4x^2 - 8x + 1$

$= -4(x^2 + 2x \qquad) + 1$

$= -4(x^2 + 2x + 1 - 1) + 1$

$= -4(x^2 + 2x + 1) + 4 + 1$

$y = -4(x + 1)^2 + 5$

$\therefore$ max. of 5

c) $y = 4x^2 + 4x + 2$

$= 4(x^2 + 1x \qquad) + 2$

$= 4(x^2 + 1x + 0.25 - 0.25) + 2$

$= 4(x^2 + 1x + 0.25) - 1 + 2$

$y = 4(x + 0.5)^2 + 1$

$\therefore$ min of 1

d) $y = \frac{1}{3}x^2 - 4x + 7$

$= \frac{1}{3}(x^2 - 12x \qquad) + 7$

$= \frac{1}{3}(x^2 - 12x + 36 - 36) + 7$

$= \frac{1}{3}(x^2 - 12x + 36) - 12 + 7$

$y = \frac{1}{3}(x - 6)^2 - 5$

$\therefore$ min of -5

Factored form to Standard form

Convert $y = 3(x + 2)(x - 1)$ to standard form.

$y = 3(x^2 - 1x + 2x - 2)$

$= 3(x^2 + 1x - 2)$

$y = 3x^2 + 3x - 6$

Factored Form to Vertex Form

Convert $y = 2(x + 1)(x - 3)$ to vertex form.

"a"

$x = -1$ or $x = 3$

Vertex

$x = \frac{-1 + 3}{2}$

$x = 1$

$y = 2(1 + 1)(1 - 3)$

$= 2(2)(-2)$

$y = -8$

$(1, -8)$

(h, k)

$\therefore y = 2(x - 1)^2 - 8$

$y = 2(x - 1)^2 - 8$

HW P214 #6ace, 7 (just complete the square), 8 (all), 16