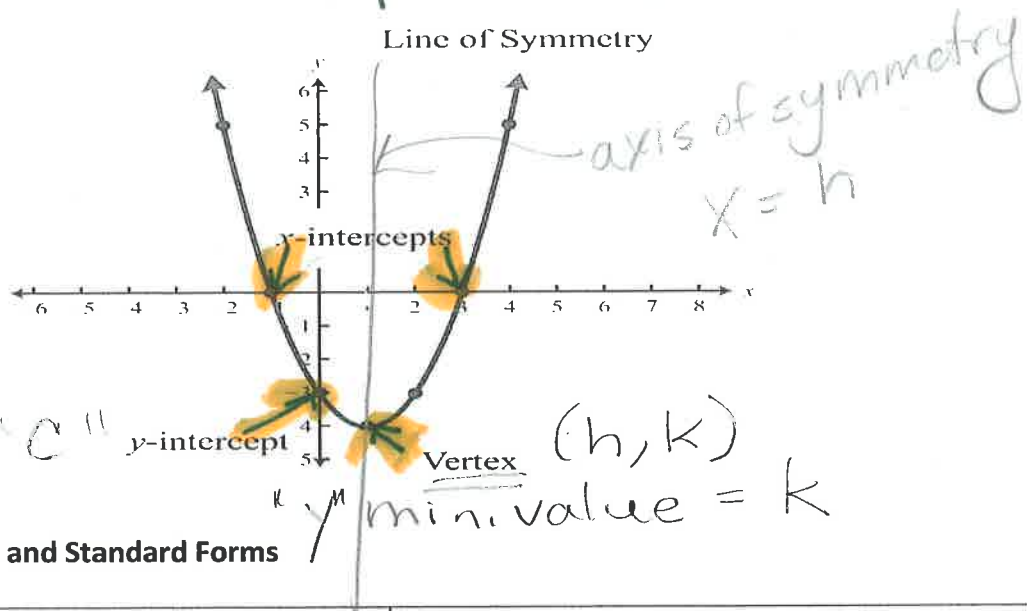


4.1 The Vertex Form of a Quadratic Function (P196 to 205)

The graph of a quadratic relation is called a parabola.

Features



Comparing Vertex and Standard Forms

Vertex Form	Standard Form
$y = a(x - h)^2 + k$	$y = ax^2 + bx + c$
<p>a <u>direction of opening, step pattern</u> (a)</p> <p>$a > 0$ opens up</p> <p>$a < 0$ opens down</p> <p>$a > 1$ or $a < -1$ stretch (narrower)</p> <p>$-1 < a < 1$ compression (wider)</p>	<p>a <u>direction of opening, step pattern</u> (a)</p> <p>Same as vertex form</p> <p>$a > 0$ opens up</p> <p>$a < 0$ opens down</p> <p>$y = x^2$ 1, 3, 5, 7</p> <p>$y = \frac{1}{2}x^2$ $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$ Steps</p>
<p>h <u>horizontal shift (translation)</u> (h)</p> <p>$h > 0$ shifts right</p> <p>$h < 0$ shifts left</p>	<p>c <u>y-intercept</u></p> <p>let $x = 0$, Solve for y</p> <p>$y = 2x^2$</p> <p>$\hookrightarrow 2, 6, 10, 14$</p>
<p>k <u>vertical shift (translation)</u> (k)</p> <p>$k > 0$ shifts up</p> <p>$k < 0$ shifts down</p>	

max/min value

= k

axis of symmetry $x = h$

Working with Vertex Form

1. Given $y = -(x + 1)^2 + 4$, determine:

$$a = -1 \quad h = -1 \quad k = 4$$

a) Direction of opening

down (-)

b) Axis of symmetry

$$x = -1 \quad (\text{shifts left by } 1)$$

c) Max / Min value

$$\text{Max } y = 4$$

d) Vertex

$$\text{Sub } x = -1$$

$$y = -(-1 + 1)^2 + 4 = 4$$

e) y-intercept

$$\text{Let } x = 0$$

$$y = -(0 + 1)^2 + 4$$

$$= -1 + 4$$

$$y = 3$$

f) x-intercepts (zeros)

$$(-1, 4)$$

$$y = -1(x + 1)(x + 1) + 4$$

$$= -1(x^2 + 2x + 1) + 4$$

$$= -x^2 - 2x - 1 + 4$$

Std. Form $y = -x^2 - 2x + 3$

$$\text{Set } y = 0$$

$$-x^2 - 2x + 3 = 0 \quad +3, -1$$

$$-1(x^2 + 2x - 3) = 0 \quad \begin{matrix} \sqrt{1} \\ -3 \end{matrix}$$

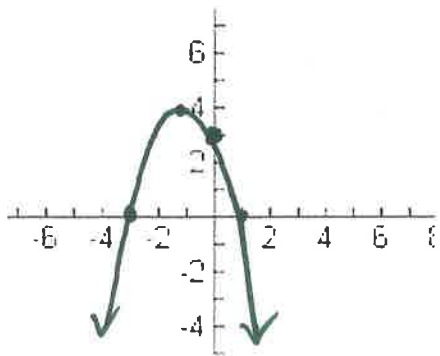
$$-1(x + 3)(x - 1) = 0$$

h) Domain and Range

$$x = -3$$

$$x = 1$$

g) Graph



$$D = \{x \mid x \in \mathbb{R}\}$$

$$R = \{y \mid y \leq 4, y \in \mathbb{R}\}$$

2. Write $y = 3(x - 4)^2 + 8$ in standard form.

$$a = 3 \quad h = 4 \quad k = 8$$

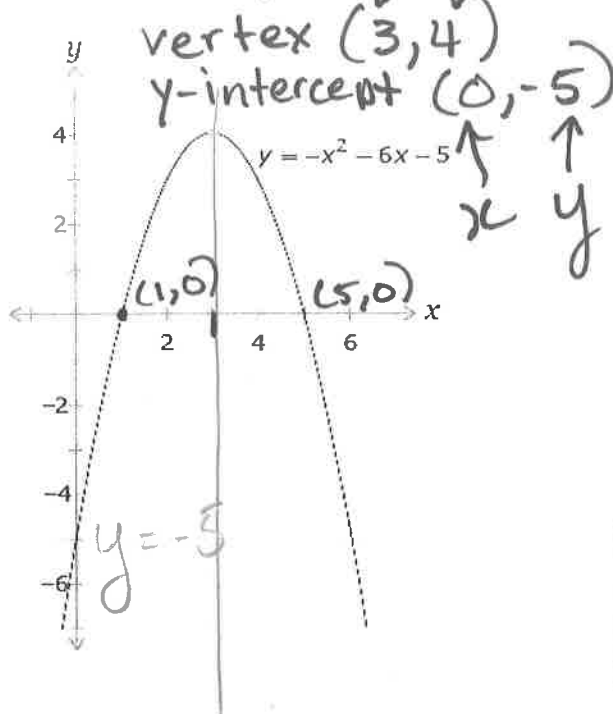
$$y = 3(x - 4)(x - 4) + 8$$

$$= 3(x^2 - 8x + 16) + 8$$

$$= 3x^2 - 24x + 48 + 8$$

$$y = 3x^2 - 24x + 56$$

3. Determine the equation in vertex form of the quadratic function shown.



$$y = a(x-h)^2 + k$$

$$-5 = a(0-3)^2 + 4$$

$$-5 = a(9) + 4$$

$$9a = -5 - 4$$

$$a = -1$$

$$y = -1(x-3)^2 + 4$$

$$x = 3$$

$$h = 3$$

4. Write the equation of the quadratic function, first in vertex form then in standard form given the vertex (3, 5) and passing through the point (1, 1).

$$= \uparrow \uparrow$$

$$h \quad k$$

$$\uparrow \uparrow$$

$$x \quad y$$

$$y = a(x-h)^2 + k$$

$$1 = a(1-3)^2 + 5$$

$$1 = a(4) + 5$$

$$4a = 1 - 5$$

$$4a = -4$$

$$a = -1$$

vertex form

$$y = -1(x-3)^2 + 5$$

$$y = -1(x-3)(x-3) + 5$$

$$= -1(x^2 - 6x + 9) + 5$$

$$= -x^2 + 6x - 9 + 5$$

$$y = -x^2 + 6x - 4$$

Standard form